

# Early Detection of Diabetic Retinopathy Using Various Techniques: A Review

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### ABSTRACT

Diabetic retinopathy is a complication of diabetes, caused by high blood sugar levels damaging the back of the eye (retina). It can cause blindness if left undiagnosed and untreated. However, it usually takes several years for diabetic retinopathy to reach a stage where it could threaten your sight. Diabetic retinopathy is caused by damage to the blood vessels in the tissue at the back of the eye (retina). Poorly controlled blood sugar is a risk factor. Early symptoms include floaters, blurriness, dark areas of vision and difficulty perceiving colors. Blindness can occur. Mild cases may be treated with careful diabetes management. Advanced cases may require laser treatment or surgery. DR is characterized by lesions on the retina and this paper focuses on detecting two of these lesions, Microaneurysms and Haemorrhages, which are also known as red lesions. Microaneurysms are usually the earliest visible manifestation of diabetic retinopathy. They appear as tiny red dots scattered in the retina posteriorly. They may be surrounded by a ring of yellow lipid, or hard, exudates or diabetic retinopathy that is threatening or affecting your sight, the main treatments are: Laser Treatment – to treat the growth of new blood vessels at the back of the eye (retina) in cases of proliferative diabetic retinopathy.

Keywords— Diabetic Retinopathy, Microaneurysms, Lesions.

### 1. Introduction

A Diabetes is a chronic disease characterized by blood glucose level elevation. This elevation leads over time to severe damage of the human blood vessels (BV), eyes, and nerves. Diabetic retinopathy (DR) is mostly one of the common complications of diabetes. It is a progressive disease that can cause permanent blindness without warning. Studies estimate that diabetes will affect about 642 million adults overall the world, while DR affects one from every three people with diabetes. Another study ensures that by 2030, the number of people with DR will grow to191 million.

### **1.1. Understanding the Eye**

The eye works in a similar way as a camera. When you look at an object, light reflected from the object enters the eyes through the pupil and is focused through the optical components within the eye.

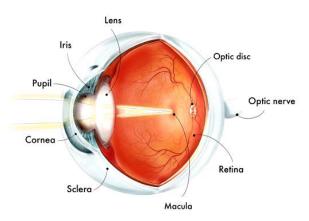


Fig 1. Anatomy of Human Eye



The front of the eye is made of the cornea, iris, pupil and lens, and they focus the image onto the retina. The retina is the light sensitive membrane that covers the back of the eye. This membrane consists of millions of nerve cells which gather together behind the eye to form a large nerve called the optic nerve.

When the light enters the eye, it is focused to a pinpoint on the macula, a small area in the center of the retina at the back of the eye. The macula is responsible for central detailed vision, allowing you to see fine detail and color, read and recognize faces.

### 1.2 When Diabetic Retinopathy Forms

People with diabetes can have an eye disease called diabetic retinopathy. This is when high blood sugar levels cause damage to blood vessels in the <u>retina</u>. These blood vessels can swell and leak. Or they can close, stopping blood from passing through. Sometimes abnormal, new blood vessels grow on the retina. All of these changes can steal vision.

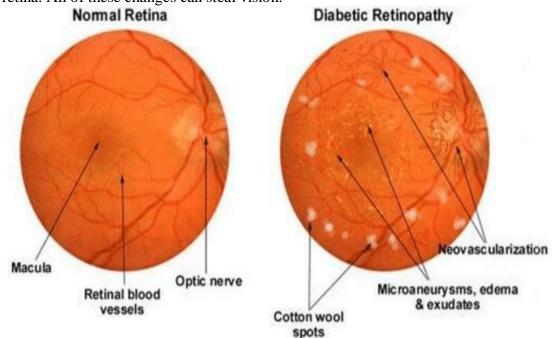


Fig 2. Difference Between Normal Retina and Diabetic Retinopathy

## **1.3 The Two Stages of Diabetic Eye Disease**

There are two main stages of diabetic eye disease.

## NPDR (non-proliferative diabetic retinopathy)

This is the early stage of diabetic eye disease. Many people with diabetes have it, with NPDR, tiny blood vessels leak, making the retina swell. When the macula swells, it is called macular edema. This is the most common reason why people with diabetes lose their vision.

Also, with NPDR, blood vessels in the retina can close off. This is called **macular ischemia**. When that happens, blood cannot reach the macula. Sometimes tiny particles called exudates can form in the retina. These can affect the vision too. NPDR can also be classified as mild, moderate and severe. If you have NPDR, your vision will be blurry.

## PDR (proliferative diabetic retinopathy)

PDR is the more advanced stage of diabetic eye disease. It happens when the retina starts growing new blood vessels. This is called neovascularization. These fragile new vessels often bleed into the vitreous. If they only bleed a little, you might see a few dark floaters. If they bleed a lot, it might block all vision. These new blood vessels can form scar tissue. Scar tissue can cause problems with the macula or lead to a detached retina.

PDR is very serious, and can steal both the central and peripheral (side) vision.

The various grades of DR (a) Normal retina anatomy, (b) Normal case, (c) Mild, (d) Moderate, (e) Severe (NPDR), and PDR.



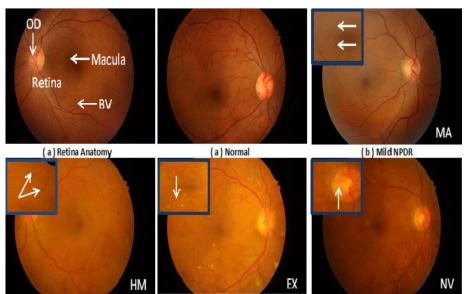


Fig 3. Various Grades of Diabetic Retinopathy

# 1.4 What Happens When You Have Diabetic Retinopathy?

Diabetic patients can have diabetic retinopathy and not know it. This is because it often has no symptoms in its early stages. As diabetic retinopathy gets worse, we will notice symptoms such as: • seeing an increasing number of floaters,

- seeing an increasing numbe
  having blurry vision,
- having binny vision,
  having vision that changes sometimes from blurry to clear,
- seeing blank or dark areas in the field of vision,
- having poor night vision, and
- noticing colors appear faded or washed out
- losing vision.

Diabetic retinopathy symptoms usually affect both eyes.

# 1.5 Diabetic Retinopathy Diagnosis by a Physician

Drops will be put in the eye to dilate (widen) your pupil. This allows the ophthalmologist to look through a special lens to see the inside of the eye. The doctor may do optical coherence tomography (OCT) to look closely at the retina. A machine scans the retina and provides detailed images of its thickness. This helps the doctor find and measure swelling of your macula.

Fluorescein angiography or OCT angiography helps the doctor see what is happening with the blood vessels in your retina. Fluorescein angiography uses a yellow dye called fluorescein, which is injected into a vein (usually in your arm). The dye travels through the blood vessels. A special camera takes photos of the retina as the dye travels throughout its blood vessels. This shows if any blood vessels are blocked or leaking fluid. It also shows if any abnormal blood vessels are growing. OCT angiography is a newer technique and does not need dye to look at the blood vessels.

# 2. Literature Survey

## 2.1 Classification of Diabetic Retinopathy Severity Based on GCA Attention Mechanism

In 2021, Mr Binhua Yang and Mr Tongyan Li Diabetic retinopathy (DR) is one of the major complications caused by diabetes and can lead to severe vision loss or even complete blindness if not diagnosed and treated in a timely manner. In this paper, a new feature map global channel attention mechanism (GCA) is proposed to solve the problem of the early detection of DR. In the GCA module, an adaptive one-dimensional convolution kernel size algorithm based on the dimension of the feature map is proposed and a deep convolutional neural network model for DR color medical image severity diagnosis named GCA-Effcient Net (GENet) is designed. The training process uses transfer learning techniques with a cosine annealing learning rate adjustment strategy. The image regions of interest



of GENet are visualized using a heat map. The accuracy, precision, sensitivity and specificity of the DR dataset of the Kaggle competition reached 0.956, 0.956, 0.956, and 0.989, respectively. A large number of experiment results show that GENet based on the GCA attention mechanism can more effectively extract lesion features and classify the severity of DR.

# **2.2** Automated Microaneurysms Detection in Retinal Images Using Radon Transform and Supervised Learning: Application to Mass Screening of Diabetic Retinopathy

In 2021, Mr Meysam Detection of red lesions in color retinal images is a critical step to prevent the development of vision loss and blindness associated with diabetic retinopathy (DR). Microaneurysms (MAs) are the most frequently observed and are usually the first lesions to appear as a consequence of DR. Therefore, their detection is necessary for mass screening of DR. However, detecting these lesions is a challenging task because of the low image contrast, and the wide variation of imaging conditions. Recently, the emergence of computer-aided diagnosis systems offers promising approaches to detect these lesions for diagnostic purposes. In this paper we focus on developing unsupervised and supervised techniques to cope intelligently with the MAs detection problem. In the first step, the retinal images are preprocessed to remove background variation in order to achieve a high level of accuracy in the detection. In the main processing step, important landmarks such as the optic nerve head and retinal vessels are detected and masked using the Radon transform (RT) and multi-overlapping windows. Finally, the MAs are detected and numbered by using a combination of RT and a supervised support vector machine classifier. The method was tested on three publicly available datasets and a local database comprising a total of 749 images. Detection performance was evaluated using sensitivity, specificity, and FROC analysis. From the image analysis viewpoint, DR was detected with a sensitivity of 100% and a specificity of 93% on average across all of these databases. Moreover, from lesion-based analysis the proposed approach detected the MAs with sensitivity of 95.7% with an average of 7 false positives per image. These results compare favorably with the best of the published results to date.

# 2.3 An Enhanced Residual U-Net for Microaneurysms and Exudates Segmentation in Fundus Images

In 2016 Mr Caixia Kou and Mr Wei Li Diabetic retinopathy (DR) is a leading cause of visual blindness. However if DR can be diagnosed and treated early, 90% of DR causing blindness can be prevented significantly. Microaneurysms (MAs) and exudates (EXs), as signs of DR, can be used for early DR diagnosis. However, MAs and EXs segmentation is a challenging task due to the low contrast of the lesions, the interference of noises, and the imbalance between the lesion areas and the background. In this paper, an enhanced residual U-Net (ERUNet) for MAs and EXs segmentation is proposed. ERU-Net obtains three U-paths, which are composed by three up sampling paths together with one down sampling path. With such three U-paths structure, ERU-Net can enhance the corresponding features fusion and capture more details of fundus images. Also, a residual block is constructed in ERU-Net to extract more representative features. In the experiments, we evaluate the performance of ERU-Net for MAs and EXs segmentation on three public datasets, E-Ophtha, IDRiD , and DDR. The ERU-Net obtains the AUC values of 0.9956, 0.9962, 0.9801, 0.9866, 0.9679, 0.9609 for MAs and EXs segmentation on these three datasets, respectively, which are greater than that of the original U-Net. Compared with some traditional methods, convolutional neural networks and other recent U-Nets, ERU-Net also performs competitively. Besides, here they have applied ERU-Net to segment optic disc (OD) on the DRISHTI-GS1 dataset, achieving the highest Jaccard index of 0.994 compared with the existing methods. The numerical results indicate that ERU-Net is a promising network for medical image segmentation.

# **2.4 Intelligent Automated Detection of Microaneurysms in Fundus Images Using Feature-Set Tuning**

In 2020, Mr Imran Usman, (Senior Member, IEEE), and Mr Khaled A. Almejali Due to the widespread of diabetes mellitus and its associated complications, a need for early detection of the leading symptoms in the masses is felt like never before. One of the earliest signs is the presence of



microaneurysms (MAs) in the fundus images. This work presents a new technique for automatic detection of MAs in color fundus images. The proposed technique utilizes Genetic Programming (GP) and a set of 28 selected features from the preprocessed fundus images in order to evolve a mathematical expression. Through the binarization of the fitness scores, the optimal expression is evolved generation by generation through a stepwise enhancement process. The best expression is then used as a classifier for real world applications. Experimental results using three publicly available datasets validate the usefulness of the proposed technique and its ability to outperform the state-of-the-art contemporary approaches.

#### 2.5 A Novel Diabetic Retinopathy Detection Approach Based on Deep Symmetric Convolutional Neural Network

In 2021, Tieyuan Liu and Mr Yi Chen, Diabetic Retinopathy (DR) may lead to blindness in diabetic patients, which is one of the most severe eye diseases. Therefore, using automatic technology to detect DR at the early phase has very vital clinical significance. In order to detect the microaneurysms (MAs) and hard exudates (HEs) of DR, a novel detection method based on deep symmetric convolutional neural network is proposed in this paper. The symmetric convolutional structure is used to improve the effectiveness of feature extraction. The proposed method also can overcome the imbalance of positive and negative samples to avoid overfitting by increasing the width and depth of the network. Furthermore, different network structures (convolution, pooling) are used to achieve different feature filtering in the stage of feature extractions. According to the experimental results, the proposed method is superior to the state-of-the-art approach on the public dataset DIARETDB1 (DB1). The detection accuracy of the objects is 92.0%, 93.2%, 93.6%, when using different filtering structures (convolution, max-pooling, ave-pooling) respectively. The detection of microaneurysms is much improved by using ave-pooling layer for feature filtering, and the max-pooling layer can improve the detection of hard exudates.

#### 2.6 Multi-Scale Attention Network for Diabetic Retinopathy Classification

In 2021, Mr Mohammad T Al-Antary and Yasmine Arafa Diabetic Retinopathy (DR) is a highly prevalent complication of diabetes mellitus, which causes lesions on the retina that affect vision which may lead to blindness if it is not detected and diagnosed early. Convolutional neural networks (CNN) are becoming the state-of-the-art approach for automatic detection of DR by using fundus images. The high-level features extracted by CNN are mostly utilized for the detection and classification of lesions on the retina. This high-level representation is capable of classifying different DR classes; however, more effective features for detecting the damages are needed. This paper proposes the multiscale attention network (MSA-Net) for DR classification. The proposed approach applies the encoder network to embed the retina image in a high-level representational space, where the combination of mid and high-level features is used to enrich the representation. Then a multi-scale feature pyramid is included to describe the retinal structure in a different locality. Furthermore, to enhance the discriminative power of the feature representation a multi-scale attention mechanism is used on top of the high-level representation. The model is trained in a standard way using the cross-entropy loss to classify the DR severity level. In parallel as an auxiliary task, the model is trained using the weakly annotated data to detect healthy and non-healthy retina images. This surrogate task helps the model to enrich its discriminative power for distinguishing the non-healthy retina images. The proposed method when implemented has achieved outstanding results on two public datasets: EyePACS and APTOS.

# **2.7 Segmenting Diabetic Retinopathy Lesions in Multispectral Images Using Low-Dimensional Spatial-Spectral Matrix Representation**

In 2019,Mr Yunlong He, Mr Wanzhen Jiao and Mr Yunfeng Shi Multispectral Imaging (MSI) provides a sequence of en-face fundus spectral slices and allows for the examination of structures and signatures throughout the thickness of retina to characterize diabetic retinopathy (DR) lesions comprehensively. Manual interpretation of MSI images is commonly conducted by qualitatively analyzing both the spatial and spectral properties of multiple spectral slices. Meanwhile, there exist



few computer based algorithms that can effectively exploit the spatial and spectral information of MSI images for the diagnosis of DR. We propose a new approach that can quantify the spatial spectral features of MSI retinal images for automatic DR lesion segmentation. It combines a generalized low-rank approximation of matrices (GLRAM) with a supervised regularization term (SRT) to generate low-dimensional spatial-spectral representations using the feature vectors in all spectral slices. Experimental results showed that the proposed approach is very effective for the segmentation of DR lesions in MSI images, which suggests it as an interesting tool for assisting ophthalmologists in diagnosing, analyzing, and managing DR lesions in MSI.

### 2.8 Automated Microaneurysms Detection in Retinal Images Using Radon Transform and Supervised Learning: Application to Mass Screening of Diabetic Retinopathy

In 2021, Mr Meysam Tavakoli and Mr Alireza Mehdizadeh Detection of red lesions in color retinal images is a critical step to prevent the development of vision loss and blindness associated with diabetic retinopathy (DR). Microaneurysms (MAs) are the most frequently observed and are usually the first lesions to appear as a consequence of DR. Therefore, their detection is necessary for mass screening of DR. However, detecting these lesions is a challenging task because of the low image contrast, and the wide variation of imaging conditions. Recently, the emergence of computer-aided diagnosis systems offers promising approaches to detect these lesions for diagnostic purposes. In this paper we focus on developing unsupervised and supervised techniques to cope intelligently with the MAs detection problem. In the first step, the retinal images are preprocessed to remove background variation in order to achieve a high level of accuracy in the detection. In the main processing step, important landmarks such as the optic nerve head and retinal vessels are detected and masked using the Radon transform (RT) and multi-overlapping windows. Finally, the MAs are detected and numbered by using a combination of RT and a supervised support vector machine classifier. The method was tested on three publicly available datasets and a local database comprising a total of 749 images. Detection performance was evaluated using sensitivity, specificity, and FROC analysis. From the image analysis viewpoint, DR was detected with a sensitivity of 100% and a specificity of 93% on average across all of these databases. Moreover, from lesion-based analysis the proposed approach detected the MAs with sensitivity of 95.7% with an average of 7 false positives per image. These results compare favourably with the best of the published results to date.

# **2.9 Robust Collaborative Learning of Patch-Level and Image-Level Annotations for Diabetic Retinopathy Grading from Fundus Image**

In 2021, Mr Yehui Yang ,Mr Fangxin Shang ,Mr Binghong Wu , Mr Dalu Yang Lei Wang and Mr Yanwu Xu Diabetic retinopathy (DR) grading from fundus images has attracted increasing interest in both academic and industrial communities. Most convolutional neural network-based algorithms treat DR grading as a classification task via image-level annotations. However, these algorithms have not fully explored the valuable information in the DR-related lesions. In this article, we present a robust framework, which collaboratively utilizes patch-level and image-level annotations, for DR severity grading. By an end-to-end optimization, this framework can bidirectionally exchange the fine-grained lesion and image-level grade information. As a result, it exploits more discriminative features for DR grading. The proposed framework shows better performance than the recent state-of-the-art algorithms and three clinical ophthalmologists with over nine years of experience. By testing on datasets of different distributions (such as label and camera), we prove that our algorithm is robust when facing image quality and distribution variations that commonly exist in real-world practice. We inspect the proposed framework through extensive ablation studies to indicate the effectiveness and necessity of each motivation. The code and some valuable annotations are now publicly available.

### 2.10 Multi-Stream Deep Neural Network for Diabetic Retinopathy Severity Classification Under a Boosting Framework

In 2022, Hamza Mustafa and Mr Syed Farooq In this paper Diabetic Retinopathy (DR) is an eye disorder in patients with diabetes. Detection of DR presence and its complications using fundus images at an early stage helps prevent its progression to the advanced levels. In the recent years,



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several well-designed Convolutional Neural Networks (CNN) have been proposed to detect the presence of DR with the help of publicly available datasets. However, these existing CNN-based classifiers focus on utilizing different architectural settings to improve the performance of detection task only i.e. presence or absence of DR. The further classification of the severity and type of the disease, however, remains a non-trivial task. To this end, we propose a multi-stream ensemble deep network to classify diabetic retinopathy severity. The proposed approach takes advantages of the deep networks and principal component analysis (PCA) to learn inter-class and intra-class variations from the raw image features. Ensemble machine learning classifiers are then applied to achieve high classification accuracy and robust performance on the obtained deep features. Specifically, a multistream network is made using pre trained deep learning architectures i.e. ResNet-50 and DenseNet-121 to serve as the main feature extractors. Further application of PCA reduces the dimensionality of features and effectively separates the variation space of inter-class and intra-class images. Finally, an ensemble machine learning classifier using AdaBoost and random forest algorithms is built to further improve classification accuracy. The proposed approach has been compared with multiple conventional CNN-based approaches on Messidor-2 (two categories) and EyePACS (two, five categories) datasets. The experiment results show that our proposed approach achieves superior performance (upto 95.58% accuracy) and can be considered a promising method for automatic diabetic retinopathy detection.

**2.11** Diabetic Retinopathy Detection Using Prognosis of Microaneurysm and Early Diagnosis System for Non-Proliferative Diabetic Retinopathy Based on Deep Learning Algorithms

In 2020, Mr Lifeng Qiao and Mr Hui Zhou, In this paper Predicting the presence of Microaneurysms in the fundus images and the identification of diabetic retinopathy in early-stage has always been a major challenge for decades. Diabetic Retinopathy (DR) is affected by prolonged high blood glucose level which leads to microvascular complications and irreversible vision loss. Microaneurysms formation and macular edema in the retinal is the initial sign of DR and diagnosis at the right time can reduce the risk of non-proliferated diabetic retinopathy. The rapid improvement of deep learning makes it gradually become an efficient technique to provide an interesting solution for medical image analysis problems. The proposed system analysis the presence of microaneurysm in fundus image using convolutional neural network algorithms that embeds deep learning as a core component accelerated with GPU(Graphics Processing Unit) which will perform medical image detection and segmentation with high-performance and low-latency inference. The semantic segmentation algorithm is utilized to classify the fundus picture as normal or infected. Semantic segmentation divides the image pixels based on their common semantic to identify the feature of microaneurysm. This provides an automated system that will assist ophthalmologists to grade the fundus images as early NPDR, moderate NPDR, and severe NPDR. The Prognosis of Microaneurysm and early diagnosis system for non - proliferative diabetic retinopathy system has been proposed that is capable to train effectively a deep convolution neural network for semantic segmentation of fundus images which can increase the efficiency and accuracy of NPDR (non proliferated diabetic retinopathy) prediction.

### CONCLUSION

This paper examined the different strategies for the DR detection. Contrasted with clinical analysis, mix of picture preparing and delicate figuring strategies yielded more precise results to identify DR. The procedure of DR detection is completed in different stages like pre-processing, feature highlighting and extraction, Classification and arrangement which utilize advanced systems for getting exact results.

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