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Some Sub-Classes of Harmonic Univalent functions

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ABSTRACT:
Complex Analysis is branch of Geometric function theory. Geometric function theory concerned with interplay between the geometric properties of the image domain and analytic properties of the mapping functions. Some properties of analytic functions are exclusive and do not extend to more general harmonic mappings. In this paper we study the some subclasses of univalent harmonic functions like Coefficient Bounds, Distortion results and Convolution of Two functions.
Keywords - Analytic functions, univalent function, harmonic functions, convex functions.

Introduction:
The most exciting element of complex function theory is probably how geometry and analysis interact. In the theory of univalent functions, these connections between geometric behavior and analytic structure are the main topics of discussion. A single valued function f(z) is said to be analytic at a point Z_0, it is differentiable at every point in some neighbourhood of Z_0. It is also known as regular or Holomorphic function. A function f(z) is said to univalent in domain D the condition $f(z_1)=f(z_2)$ implies $z_1 = z_2$ where $z_1, z_2 \in D$

Definition 1.1: Class A:
Let A be the class of all analytic normalized functions f in the open unit disk E = {z: |z|< 1} with normalized conditions f (0) = 0 and $f^1(0) = 1$, having a Taylor's series expansion of the form

$$f(z) = z + \sum_{n\geq 2}^{\infty} a_n z^n$$

Definition 1.2: Class S
The subfamily of A denoted by S consists of all simple functions like z, z/1-z…… are some of familiar functions of the class S. The class s is well known to be closed under several operations like rotation, conjugation, dilation, range transformation, disc automorphism, square root transformation etc.

Definition 1.3: Class $\mathcal{S}_{\mathcal{H}}$:
The continuous function $f = u + iv$ defined in a domain $\Omega \subseteq \mathbb{C}$ is harmonic in Ω, if u and v are real harmonic in Ω. In any simply connected domain Ω, we can write

$$f = h + \bar{g}$$

$$h(z) = z + \sum_{n=0}^{\infty} a_n z^n$$

and

$$g(z) = z + \sum_{n=1}^{\infty} a_n z^n$$

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Where h and g are analytic. We call h as the analytic part and g as the co-analytic part of f. Due to Lewy [1], the Jacobian off is then given by

$$J_f(z) = |h'(z)|^2 - |g'(z)|^2$$

When J_f is positive in Ω, the harmonic function f is called orientation - preserving or sense preserving in Ω.

Definition1.4: A function $f \in \mathcal{H}$ given by (1) is said to be in the class $\mathcal{S}_H^\lambda[\alpha,\beta]$ with α,β∈ [0,1) and $0 < \lambda < \infty$ f is the analytic part of f is a member of $\mathcal{S}^\lambda[\alpha]$ and $|b_1| = \beta$ Equivalently,

$$\mathcal{S}_H^\lambda[\alpha,\beta] := \left\{ \begin{matrix} f = h + \bar{g} \in \mathcal{H} : h(z) \in \mathcal{S}^\lambda[\alpha], |b_1| = \beta; \\ \alpha, \beta \in [0,1); 0 < \lambda < \infty \end{matrix} \right\}$$

Lemma 1([11]). If $\Phi(z) = c_0 + c_1 z + c_2 z^2 + \cdots$ is an analytic function and $|\Phi(z)| \leq 1$ on the open unit disk $\mathbb{D}$ then,

$$|c_n| \leq 1 - |c_0|^2, n = 1,2,3, \ldots$$

Lemma 2([20]). If the function $\in \mathcal{S}^\lambda[\alpha]$, then

$$|a_n| \leq \frac{(1-\alpha)}{n^\lambda(n-\alpha)}, n = 2,3, \ldots$$

and equality holds for each n only for functions of the form

$$f_n(z) = z + \frac{(1-\alpha)}{n^\lambda(n-\alpha)} e^{i\theta} z^n, \theta \in \mathbb{R}, z \in \mathbb{D}.$$

Definition1.5 Convolution (or Hadamard product):
For analytic functions, $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and
$F(z) = z + \sum_{n=2}^{\infty} A_n z^n$, their convolution or Hadamard product, denoted by $f * F$, is defined as

$$(f * F)(z) = z + \sum_{n=2}^{\infty} a_n A_n z^n.$$

Theorem 1.1 (The Riemann mapping theorem). Let D be a simply connected domain in C with D ∈ C and let z_0 be a point in D. Then there exists a unique mapping D onto the unit disc D = {z ∈ C : |z| < 1} which is analytic and injective in D with $f(z_0) = 0$ and $f^1(z_0) > 0$.

Main Result:

Let A denote the class of functions f of the form of $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$ which are analytic in open $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ The subclass of A consisting of all analytic and univalent functions in D will be denoted by S. A well-known sufficient condition for a function to be in the class S is that $\sum_{n=2}^{\infty} n|a_n| \leq 1$ An analogous sufficient condition for a function f to be in the class $\mathcal{S}^\lambda[\alpha], 0 \leq \alpha < 1, \quad 0 < \lambda < \infty$ is that

$$\sum_{n=2}^{\infty} n^\lambda \left(\frac{n-\alpha}{1-\alpha}\right) |a_n| \leq 1$$

Note that for each fixed n the function nλ is increasing with respect to λ. This shows that if λ increases, t. Consequently, the functions in $\mathcal{S}^\lambda[\alpha]$ are univalent starlike of order α if λ≥0 and if λ≥1 then the functions in the family $\mathcal{S}^\lambda[\alpha]$ are univalent convex of order α.

A complex-valued harmonic function is a simply connected domain D subset of the complex plane C has a representation $f = h + \bar{g}$ where h and g are analytic functions in D, that is unique up to an additive constant. The representation $f = h + \bar{g}$ is therefore unique and is called the canonical representation of f. Lewy in [11] proved that f is locally univalent if and only if the Jacobian satisfies $J_f = |h'|^2 - |g'|^2 \neq 0$ thus, harmonic mappings are either sense-preserving or sense-reversing depending on the conditions $J_f > 0$ and $J_f < 0$, respectively throughout the domain D, where f is locally univalent. Since $J_f > 0$ if and only if $J_f < 0$ we will consider sense-preserving mappings in $\mathbb{D}$ throughout all of this work. In this case the analytic part h is locally univalent in D since h′≠0, and the second complex dilatation w of f $= g'/h'$, is an analytic function in D with $|w| < 1$, see [12].

Let $\mathcal{H}$ denote the set of all locally univalent and sense preserving complex harmonic mappings in D. Therefore, any function f in the class $\mathcal{H}$ has unique power series representation of the form:

(1) $f = h + \bar{g}$ Where $h(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=1}^{\infty} b_n z^n (z \in \mathbb{D})$

And $a_n, b_n \in \mathbb{C}$ Following Clunie and Sheil-Small's notation [10],

let $\mathcal{S}_{\mathcal{H}} \subset \mathcal{H}$ denote the class of all sense preserving univalent harmonic functions $f = h + \bar{g}$ and $\mathbb{D}$ with the normalization $h(0) = g(0) = h'(0) - 1 = 0$. The class $\mathcal{S}_{\mathcal{H}}$ is a normal family [13].

It is pertinent that for a fixed analytic function h an interesting problem arises to describe all functions g such that f ∈ H. Not much known on the geometric properties of such planar harmonic functions. Klimek and Michalski [14], first studied the properties of a subset of SH which is defined for all univalent anti-analytic perturbation of the identity and also considered the subclass of SH which is defined by restricting h as a member of C, univalent convex functions [15]. Very recently, Hotta and Michalski [13] considered h as a member of S *, univalent star like functions and discuss some geometric properties of certain subfamily of S_H. Few more subclasses of planar harmonic mappings were considered by restricting h as member of univalent star like function of order α [16], univalent convex function of order α [16] and to be in the class of bounded boundary rotation [17].

1. Coefficient Bound:

Coefficient bound. In this section we have studied the bound of $|b_n|$, for $= h + \bar{g} \in \mathcal{S}_H^{\lambda}[\alpha, \beta]$, with $\lambda \geq 0$ where h and g have the series representations of the form (1).

Theorem1.2:

Let $f = h + \bar{g} \in \mathcal{S}_H^{\lambda}[\alpha, \beta], \geq 0$ where $h(z)$ and $g(z)$ are given by (1). Then

$$(2) \quad |b_n| \leq \begin{cases} \frac{(1-\alpha)\beta}{2^{\lambda}(2-\alpha)} + \frac{(1-\beta^2)}{2}, & n = 2 \\ \frac{(1-\alpha)(1-\beta^2)}{n} \sum_{k=1}^{n-1} \frac{k^{1-\lambda}}{k-\alpha} + \frac{(1-\alpha)\beta}{n^{\lambda}(n-\alpha)}, & n = 3,4,\dots \end{cases}$$

Proof:

Let the function $f(z) = h(z) + \overline{g(z)}$ be in the class $\mathcal{S}_H^{\lambda}[\alpha, \beta]$ where h and g are represented by (1). h and g are represented by (1). Let $g'(z) = w(z)h'(z)$ where $w(z)$ is the dilatation of *f*.

$$(3) \qquad w(z) = \sum_{n=0}^{\infty} c_n z^n (z \in \mathbb{D})$$

where $c_n \in \mathbb{C}$. Clearly, $c_0 = |w(0)| = |g'(0)| = |b_1| = \beta < 1$. Further, since $f \in \mathcal{S}_H^{\lambda}[\alpha, \beta]$ is sense preserving, we have $|w(z)| < 1$ for all $z \in \mathbb{D}$ .Therefore from Lemma 1.2, we have

$$|c_n| \leq 1 - |c_0|^2, n = 1,2, \dots$$

simplifying $g'(z) = w(z)h'(z)$, by using relations (1) and (3) we have

$$(4) \qquad \sum_{n=1}^{\infty} n b_n z^{n-1} = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n-1} (k+1) a_{k+1} c_{n-k-1} \right) z^{n-1}.$$

Comparing the coefficients in (4), we get

$$(5) \qquad n b_n = \sum_{k=0}^{n-1} (k+1) a_{k+1} c_{n-1-k}, n = 2,3, \dots$$

Since $h(z) \in \mathcal{S}_H^{\lambda}[\alpha]$ it is clear from Lemma 2 that

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(6) $$|a_n| \leq \frac{1-\alpha}{n^\lambda(n-\alpha)}, n = 2,3,4,\ldots$$

Application of (6) and (5) together with Lemma 1 gives

$$n|b_n| \leq \sum_{k=0}^{n-2} (k+1)|a_{k+1}||c_{n-1-k}| + n|a_n||c_0|$$
$$\leq \sum_{k=0}^{n-2} \frac{(k+1)(1-\alpha)(1-\beta^2)}{(k+1)^\lambda(k+1-\alpha)} + \frac{n(1-\alpha)\beta}{n^\lambda(n-\alpha)}$$

which implies that

$$|b_n| \leq \frac{(1-\alpha)(1-\beta^2)}{n}\sum_{k=1}^{n-1} \frac{k^{1-\lambda}}{k-\alpha} + \frac{(1-\alpha)\beta}{n^\lambda(n-\alpha)}$$

In particular for $n=2$, we have

$$2|b_2| \leq 2|a_2||c_0| + |a_1||c_1| \leq \frac{2(1-\alpha)\beta}{2^\lambda(2-\alpha)} + 1 - \beta^2,$$

which together with Lemma 1 and Lemma 2 provides

$$|b_2| \leq \frac{(1-\alpha)\beta}{(2-\alpha)2^\lambda} + \frac{(1-\beta^2)}{2}$$

2.Distortionresult:

In this section, we found the growth and distortion estimates of the analytic and co-analytic part of function f in the class $\mathcal{S}_H^\lambda[\alpha,\beta]$.

Theorem1.3:

Let $f(z) = h(z) + \overline{g(z)} \in \mathcal{S}_H^\lambda[\alpha,\beta], \lambda \geq 0$, where $h(z)$ and $g(z)$ are given by (1). Then for $z = re^{i\theta}, \theta \in \mathbb{R}$,wehave

(7) $$1 - \frac{(1-\alpha)r}{(2-\alpha)2^{\lambda-1}} \leq |h'(z)| \leq 1 + \frac{(1-\alpha)r}{(2-\alpha)2^{\lambda-1}}$$

and

(8) $$\left(\frac{|\beta - r|}{1-\beta r}\right)\left(1 - \frac{(1-\alpha)r}{(2-\alpha)2^{\lambda-1}}\right) \leq |g'(z)| \leq \left(\frac{\beta + r}{1+\beta r}\right)\left(1 + \frac{(1-\alpha)r}{(2-\alpha)2^{\lambda-1}}\right).$$

Proof.

Let $g'(0) = \beta e^{i\mu}, \mu$ real. From a given dilatation $w(z), |w(z)| < 1$ and $|w(0)| = |g'(0)| = \beta$, we consider

$$F_0(z) := \frac{e^{-i\mu}w(z) - \beta}{1 - \beta e^{-i\mu}w(z)} = e^{-i\mu}\left(\frac{w(z) - \beta e^{i\mu}}{1 - \beta e^{-i\mu}w(z)}\right), \ (z \in \mathbb{D}).$$

Since $\beta \in [0,1)$, therefore the complex conjugate of $\beta e^{i\mu}$ is equal to $\beta e^{-i\mu}$. Further as $|w(z)| < 1$ and $|\beta e^{i\mu}| < 1$, clearly, we have $|F_0(z)| < 1$. Therefore, $F_0(z)$ satisfies the conditions of Schwartz lemma. Hence $|F_0(z)| \leq |z|$. This implies that

$$|e^{-i\mu}w(z) - \beta| \leq |z||1 - \beta e^{-i\mu}w(z)|, \ (z \in \mathbb{D})$$

which is equivalent to

(9) $$\left|e^{-i\mu}w(z) - \frac{\beta(1-r^2)}{1-\beta^2r^2}\right| \leq \frac{r(1-\beta^2)}{(1-\beta^2r^2)}, \ \left(z = re^{i\theta} \in \mathbb{D}\right).$$

Equality in the above inequality holds for the function

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$$(10) \qquad w(z) = e^{i\mu}\frac{e^{i\phi}z+\beta}{1+\beta e^{i\phi}z},\ (z\in\mathbb{D},\ \phi\in\mathbb{R}).$$

Applying triangle inequality over (10), we obtain

$$(11) \qquad \frac{|\beta-r|}{1-\beta r} \le |w(z)| \le \frac{\beta+r}{1+\beta r},\ |z|=r<1.$$

The function $f = h+\bar{g} \in \mathcal{S}_H^{\lambda}[\alpha,\beta]$, indicates that $h(z)\in S^{\lambda}[\alpha]$. Hence,

$$(12) \qquad \sum_{n=2}^{\infty} n^{\lambda}\left(\frac{n-\alpha}{1-\alpha}\right)|a_n| \le 1.$$

Clearly,

$$(2-\alpha)\sum_{n=2}^{\infty} n^{\lambda}|a_n| \le \sum_{n=2}^{\infty} n^{\lambda}(n-\alpha)|a_n| \le (1-\alpha)$$

Therefore,

$$(13) \qquad \sum_{n=2}^{\infty} n^{\lambda}|a_n| \le \frac{1-\alpha}{2-\alpha}$$

For $\delta \ge 0, n^{\lambda}$ is increasing in n. Thus using (12) and (13), we get

$$2^{\lambda}\sum_{n=2}^{\infty} n|a_n| \le \sum_{n=2}^{\infty} n^{\lambda}n|a_n| \quad = \sum_{n=2}^{\infty} n^{\lambda}(n-\alpha)|a_n| + \sum_{n=2}^{\infty}\alpha n^{\lambda}|a_n|$$
$$\le (1-\alpha)+\alpha\left(\frac{1-\alpha}{2-\alpha}\right).$$

Therefore,

$$(14) \qquad \sum_{n=2}^{\infty} n|a_n| \le \frac{1-\alpha}{2^{\lambda-1}(2-\alpha)}$$

Consider the function

$$G(z) := zh'(z) = z+\sum_{n=2}^{\infty} na_n z^n\ (z\in\mathbb{D}).$$

Therefore, using (14), we have

$$|G(z)| = |zh'(z)| \le |z| + \sum_{n=2}^{\infty} n|a_n||z|^n \le r + r^2\frac{1-\alpha}{2^{\lambda-1}(2-\alpha)}$$

Which gives the right hand side of the equality
Similarly,

$$|G(z)| = |zh'(z)| \ge |z| - \sum_{n=2}^{\infty} n|a_n||z|^n \ge r - r^2\frac{1-\alpha}{2^{\lambda-1}(2-\alpha)},$$

Which gives the left hand side of the inequality (7)

By using (11) and (7), in the identity $g'(z) = w(z)h'(z)$,
we have (15) $|g'(z)| \le \left(\frac{\beta+r}{1+\beta r}\right)|h'(z)| \le \left(\frac{\beta+r}{1+\beta r}\right)\left(1+\frac{(1-\alpha)r}{(2-\alpha)2^{\lambda-1}}\right)$,
and

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$$|g'(z)| \geq \frac{|\beta - r|}{1-\beta r}|h'(z)| \geq \left(\frac{|\beta - r|}{1-\beta r}\right)\left(1 - \frac{(1-\alpha)r}{(2-\alpha)2^{\delta-1}}\right) \quad (16)$$

This concludes the proof of this theorem.

3. Convolution of Two Functions:

The convolution of two functions of form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \text{ and } F(z) = z + \sum_{k=2}^{\infty} A_k z^k \text{ is defined as}$$

$$(f * F)(z) = f(z) * F(z) = z + \sum_{k=2}^{\infty} a_k A_k z^k$$

The integral convolution is defined by

$$(f \diamond F)(z) = z + \sum_{k=2}^{\infty} \frac{a_k A_k z^k}{k}$$

Recently W.G. Atshan et.al [18] and K.K. Dixit et.al [19] have defined and studied a subclass of harmonic univalent functions using integral convolution. They have studied the coefficient estimates, extreme points, convex combination, convolution, Bernardi and J-KimSrivastava operators.

4. Convolution (Hadamard Product) :

Define the convolution of two harmonic functions of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k + \sum_{k=1}^{\infty} \overline{\boldsymbol{b_k}} \bar{z}^k \text{ and}$$

$$\mathrm{F}(z) = z + \sum_{k=2}^{\infty} c_k z^k + \sum_{k=1}^{\infty} \overline{\boldsymbol{d_k}} \bar{z}^k.$$

We define the convolution of two harmonic functions f and F as

$$(f * \mathrm{F})(z) = f(z) * \mathrm{F}(z) = z + \textstyle\sum_{k=2}^{\infty} a_k c_k z^k + \sum_{k=1}^{\infty} \overline{\boldsymbol{b_k}}\, \overline{\boldsymbol{d_k}} \bar{z}^k.$$

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