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Optimal Timetable Formation Problem

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ABSTRACT

We consider the following problem. Given a set of m agents and n time slots for performing jobs such that agent i has a preference value of p_{ij} on slot j for each i and j. Our task is to find an optimal time table which maximises the minimum total preferences of all the agents, thereby ensuring the maximum fairness in the time slot allocation among the agents. We show that this problem is NP - complete and has a polynomial time randomised algorithm that computes the optimal solution of the problem with very high probability that every time slot has been allocated to at most agent and every agent receives time slot bundles of optimum size. The crux of our solution relies on solving two linear programs namely Slot Allocation LP and Slot Bundle LP followed by a rondomised slot allocation which ensures the high probability of the method in getting the optimal solution.

Keywords—Optimal Timetable, Slot Allocation LP, Slot Bundle LP

1. Introduction

Resource allocation problems are one of the classical NP hard problems in Computational Complexity. This category of problems has immensely large applications, including the optimal resource allotment in a multiprocessing environment to resource allotment in small Embedded Tasks. Among these problems, one of the most important category of problems is the *Max-Min Fairness* problems, wherein we attempt to find an allocation of the resources in such a way as to maximize the minimum value of some parameter among all possible allocations of the resources. Optimal Timetable Formation problem is one such problem where we are given a set A of m agents, n time slots and a mn tuple P whose $(i, j)^{th}$ entry represents the preference value P_{ij} of an agent i in availing the time slot j. We are asked to find an allocation of the time slots among the agents in such a way that the minimum total preference value of all the agents is maximized. In other words, we are required to partition the set R into m blocks $R_1, R_2, R_3, \ldots, R_m$ where R_i is the set of time slots to be allocated to the agent i in such way that $min\left\{\sum_{j\in R_i} P_{ij} \middle| i \in A\right\}$ attains the maximum possible value.

This problem has been studied in literature with different names [1][2][3][7]. The task of scheduling jobs on machines can be considered to be similar to this problem, wherein we are asked to allocate jobs to machines with a view to maximize the fairness value. This problem is shown to be NP-Complete [1] and it had been shown that the task of finding an optimal schedule for the general case is still difficult [1]. Hence, most of the researches focus on solving some special cases of the problem. Among these special cases, the one that is studied mostly in the literature is when P_{ij} can either be 0 (meaning that the agent i is not interested in slot j) or has a preference P_j (means that the agent i has agent independent preference on slot j). Several Approximation algorithms had been developed for this version of the problem [1].

1.1 Motivation for the work

The work is motivated by the fact that there are several scenarios in real life where we need to address the time slot allocation problems with maximum possible satisfactions among the underlying entities. For example, such a situation arises in institutions where the courses are to be allocated among the faculty in such a way as to give the maximum possible satisfaction among the faculty or when the class timetables are to prepared taking into account the preferences of the faculty. Therefore, a study on forming an algorithm for such situations seems to be extremely useful.

1.2 Our Contributions

We essentially prove the following results.

Theorem 1.

Optimal Timetable formation problem is NP-complete.

Hence, our research was to attempt randomized algorithms for solving the problem. We use linear programs namely *Slot Allocation LP* and *Slot Bundle LP* to solve the problem, whose relaxation can easily be solved using the simplex methods[4]. Considering the values of the decision variables in the optimal solution of the LP relaxation as probability values, we attain the following result.

Theorem 2.

There exists a polynomial time randomized algorithm for solving the optimal timetable formation problem.

2. Hardness of the Problem

In this section, we prove that the problem is NP-complete [5] by proving the following Theorem.

Theorem 1.

Optimal Timetable formation problem is NP-Complete.

Proof.

It is easy to see that the problem belongs to the class NP since checking whether a given certificate consisting of an allocation of time slots to the agents yields a given fairness value or not can be done in polynomial time. To prove the NP-hardness of the problem, we give a polynomial time reduction of the Santa Claus problem to the Optimal timetable formation problem. Given a Santa Claus instance consisting of m kids, n gifts with predefined happiness values, form an optimal Timetable formation instance by taking the kids as the agents, time slots as the gifts and the happiness values as the predefined preference values. We see that the Santa Claus instance has an allocation giving rise to an allocation of minimization happiness T if and only if the optimal timetable formation instance has a solution with a value of T. Hence, the Theorem. □

3 The Randomised Algorithm

We formulate the algorithm by means of two linear programs namely *Slot Allocation LP* and *Slot Bundle LP* which are described below.

Let ***In*** be an input instance of the Optimal timetable formation problem, which consists of m agents and n time slots. Then the Slot Allocation LP for In is given by

$$\begin{aligned}
&\text{maximize} \quad \mu \\
&\text{subject to} \\
&\sum_{i \in A} x_{ij} \leq 1 \quad \forall j \in R \\
&\sum_{j \in R} p_{ij} x_{ij} \geq \mu \quad \forall i \in A \\
&0 \leq x_{ij} \leq 1 \qquad \forall i \in A, \forall j \in R
\end{aligned}$$

Figure 1: Slot Allocation LP

A bundle of preference μ for an agent refers to a collection of time slots such that the sum of the preference values of all the time slots in the bundle is at least μ. The set of bundles of preference μ for an agent i is denoted by $B(i, \mu)$. Now the Slot Bundle LP is given by

$$
\begin{aligned}
&\text{maximize} \quad 0\\
&\text{subject to}\\
&\sum_{C \in B(i,\mu)} x_{iC} \geq 1 \quad \forall i \in A\\
&\sum_{i \in A} \left(\sum_{\substack{C: C \in B(i,\mu)\\ j \in C}} x_{iC} \right) \leq 1 \quad \forall j \in R\\
&0 \leq x_{iC} \leq 1 \qquad \forall i \in A, \forall C \in B(i,\mu)
\end{aligned}
$$

Figure 2: Slot Bundle LP

It is not hard to see that any feasible solution of the Slot Bundle lP can be converted to some feasible solution $[y_{ij} | i \in A, j \in R]$ of the Slot allocation LP as $y_{ij} = \sum_{C \in B(i,\mu)} x_{iC}$.

The randomised algorithm that computes the optimal solution of the optimal timetable formation problem is given below. The algorithm takes the instance In of the optimal timetable formation problem and solve the Slot bundle LP on ***In*** until the largest bundle size μ_0 is found out. This can be done buy repeatedly using the binary search by varying the value of μ starting from the minimum preference value. The algorithm makes use of the corresponding feasible solution to compute the feasible solution of the Slot Allocation LP as desribed above. Then the algorithm finds the desired solution of by assigning the time slot *j* to agent *i* with a probability p_{ij}.

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Algorithm 1: Algorithm to solve Optimal Time Table Formation Problem
  Data: Optimal Time Table Formation instance $In$
  Result: A solution of $In$
  Starting from $\mu$ to be the smallest resource utility, solve the Slot Bundle LP using binary
    search until the highest possible value of $\mu$ is found out. Let this value of $\mu$ be $\mu_0$;
  Let $(x_{iC} | i \in A, C \in B(I, \mu_0))$ be the solution of Slot Bundle LP for $\mu = \mu_0$. Convert this
    solution to the solution $[y_{ij} | i \in A, j \in R]$ of Slot Allocation LP;
  For each agent $i$ and for each time slot $j$, obtain the solution $[u_{ij} | i \in A, j \in R]$ by taking
    $u_{ij}$ as 1 or 0 with a probability $y_{ij}$;
  Return the solution $[u_{ij} | i \in A, j \in R]$;
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Lemma 4.1. *Algorithm 1 solves the Optimal Time Table formation problem in polynomial time such that each agent gets time slots of total preference* μ_0 *with a probability of at least 0.63369 and each time slot gets assigned to exactly one agent with a probability of at least* $y_{min} |A| e^{y_{min} - 1}$ *where* y_{min} *is the least value of the* y_{ij} *values.*

Proof. Let Z_i be the random variable which is assigned to 1 or 0 according to whether agent i receives a configuration of size μ_0 .

Now

$$\begin{aligned}\Pr(Z_i = 0) &= \prod_{C \in B(i,\tau_0)} (1 - x_{iC}) \\ &\geq \prod_{C \in B(i,\tau_0)} e^{-xiC} \\ &= e^{-\sum_{C \in B(i,\tau_0)} x_{iC}} \\ &= e^{-1}\end{aligned}$$

Hence $\Pr(Z_i = 1) = 1 - e^{-1} = 0.63369$

Let W_j be the random variable that is assigned 1 or 0 according to whether job j is allocated to exactly one agent or not. Now

$$\begin{aligned}\Pr(W_j = 1) &= \sum_{i, \in A} y_{ij} \prod_{\substack{i \in A \\ i \neq i'}} (1 - y_{i'j}) \\ &= \sum_{i \in A} y_{ij} e^{y_{ij} - 1} \\ &\geq y_{min} |A| e^{y_{min} - 1}\end{aligned}$$

Hence the Lemma. □

It is possible to achieve the desired level of precision and probability by executing the algorithm desired number of times and then taking the combined probability. Now the Theorem easily follows from Lemma 4.1.

Theorem 2. *There exists a polynomial time randomized algorithm for solving the optimal timetable formation problem.*

4. Conclusion

In the preceding sections, we had seen how a randomised algorithm can be designed for the Optimal Timetable formation problem. There are several improvements which can be made to the algorithm so that the probability of a time slot getting allocated to exactly one agent is high. An alternative for solving the problem is to develop approximation algorithms for the problem. A nice handle for such a direction can be made through the two linear programs we introduced here. On designing appropriate rounding strategies [6], we can round the fractional solutions of both the linear programs to the solutions of the problem with constant approximation ratio.

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