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MATHEMATICAL MODEL AND FEA ANALYSIS OF PROSTHETIC KNEE JOINT

Chandrakant N.Pujar[1], Prof. S.J. Sanjay[2], Dr.C. Shashishekar[3]
[1]*P.G. Student, Mechanical Engineering Department, Basaveshwar Engineering College, Karnataka, India*
[2]*Associate Professor, Department of Mechanical Engineering, Siddaganga Institute of Technology, Tumakuru*
[3]*Research Scholar, Department of Mechanical Engineering, Siddaganga Institute of Technology, Tumakuru*
Author Orcid ID: 0009-0001-3107-9853,0000-0003-3013-2795,0000-0003-2539-4459

Abstract
One of the challenges in creating a 3D human knee joint model is accurately defining the area of interest. Prior studies employed simplified bone models, which could potentially compromise the precision of the analysis, posing a contemporary issue. To address this, a multi-step technique was adopted to mitigate the problem and enhance the accuracy of the human knee joint model using analytical and simulation methods. In this endeavour, healthy male knee scanning data were used to reconstruct 3D knee models. Various types of units were employed to segment different parts of the bones. The knee model comprised distinct bone components, and cartilages were created by removing and distinguishing bone layers. For representation, the knee models of a person in both a standing position and while ascending stairs were represented by linear spring elements. Extensive analysis calculations were conducted to validate these models.
Keywords—Simulation method, Analytical method, Cartilages, Bone Layers.

I. Introduction
The knee joint is a vital weight-bearing joint located in the lower extremities, and it ranks among the largest and most complex joints in the body. Positioned between the proximal and distal segments of bones, the knee allows for a wide range of movements, including internal-external rotation, varus-valgus angulation, medial-lateral flexion-extension, lateral, proximal-distal, and anterior-posterior motions [1].

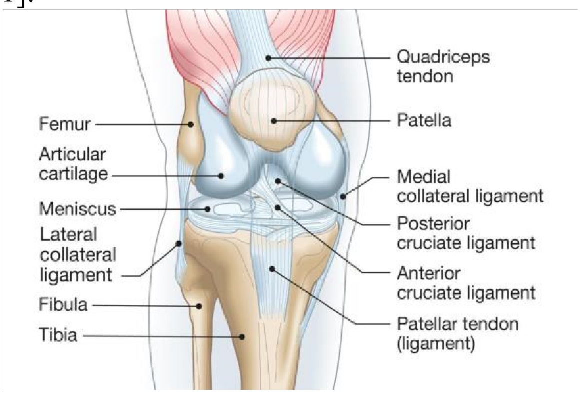

Fig 1. Natural human knee joint.

Simulation models of human bones and joints have gained widespread recognition and use in biomechanical analysis due to their ability to provide insights into biomechanical behaviors under various configurations and loads. This approach allows for the prediction and measurement of local parameters, including internal normal stress and Hertz contact stress, enhancing our understanding of these critical factors in knee joint mechanics.[2]

A. Knee joint Prosthesis

The development of a prosthetic knee joint requires careful consideration of three key factors: the distal part of the femur, the tibial surface, and the posterior aspect of the patella. Due to the bending that occurs at the end, it is recommended to use metal components for the femoral end. A patellar component is introduced to facilitate smooth motion between the two components of the joint. The upper portion of the tibia is constructed with a combination of high quality metal and plastic composites, designed to withstand weight-bearing loads and extend the longevity of the implant.[3]

B. Knee Replacement

A prosthesis is an artificial device employed to substitute any body part that is either absent or impaired. In the field of medicine, the term "missing body part" pertains to any section of the body that is no longer present due to accidents,

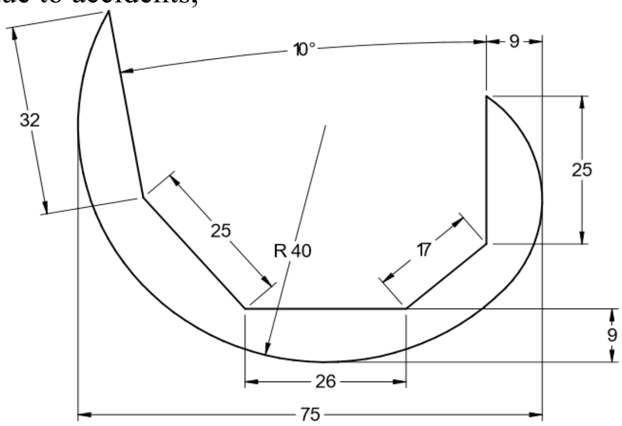

Fig 2. Mathematical model for 40mm radius and different angles

illnesses, or genetic abnormalities. There are primarily two categories of prostheses: somatoform (related to the body) and craniofacial.Unlike somatoform prostheses, which encompass replacements for limbs, ears, and other malfunctioning body parts, craniofacial prostheses involve artificial organs designed to address issues with limbs, ears, and other non-functional body parts.[4]

II. Procedure for Preparation of Mathematical model of prosthetic Knee Joint [5]

A. Making an incision in the knee region.

The surgical process entails the surgeon making an incision along the front part of your knee to access the patella, which is commonly referred to as the kneecap. In traditional knee replacement surgery, this incision is typically about 8 to 10 inches long. On the other hand, minimally invasive knee surgery involves a shorter incision, usually around 4 to 6 inches in length. There is an ongoing discussion about whether the advantages of a smaller scar outweigh the disadvantages of having a more limited surgical field.

B. Turning or repositioning the kneecap (patella).

The first step in exposing your knee involves revealing the patella, commonly referred to as the kneecap. Once the knee is opened, the surgeon relocates the patella away from the knee area. This manoeuvre allows the surgeon to obtain a clear view of the essential area required for performing the surgical procedure.

C. Preparing the femur (thighbone).

The first bone to undergo reshaping by your surgeon is the femur, also known as the thighbone. Once the knee joint is exposed after being opened up, the surgeon will carefully measure your bones and make precise cuts using specialized instruments.

D. Implanting the femoral component.

The surgeon attaches the metal femoral component to the end of your femur and secures it in place using bone cement.

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III. Material Properties

Table no 2. Material Properties

		Notation	Unit
Material Name	Epoxy resin		
Density	1.3	ρ	Kg/m3
Poisson's ratio	0.34	υ	
Young's Modulus	3.35	E	MPa
Young's Modulus for contact stress	30	E1 & E2	MPa
Poisson's ratio for contact stress	0.46	$\upsilon 1$ & $\upsilon 2$	

IV. Analytical Method

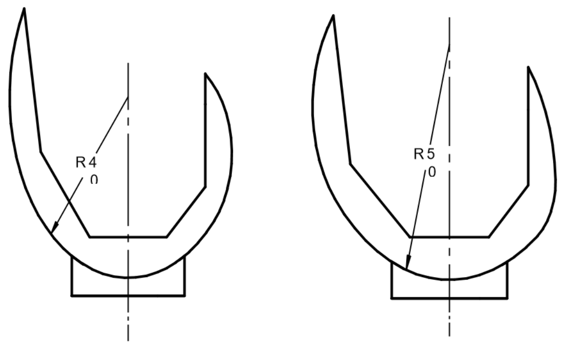

Fig 3. Mathematical model of different dimensions.

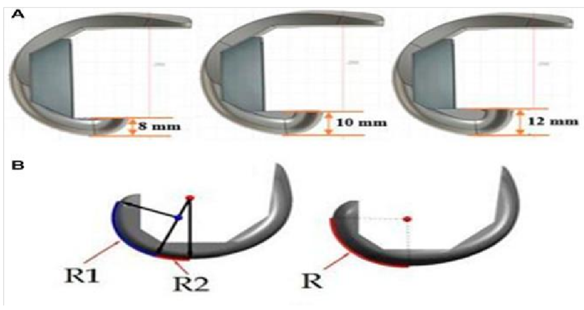

Fig 4. 3D Model of knee pan.

C. Calculation of Normal and Contact Stress [6]

Case 1. Standing stress on the knee.

The stress experienced on the tibial portion when standing can be characterized as...

$$\sigma = \frac{force\ acting\ on\ the\ \ knee\ while\ standing}{surface\ Area}$$

$$\sigma = \frac{\mathrm{F}}{\mathrm{A}}$$

Stresses are applied to the knee when an individual is in a standing position or when they are going up and down a staircase. We will now examine the stresses that affect the tibial surface, with the

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tibial surface having an area denoted as A = 2298 mm2. The largest contact area is experienced when in the standing position. Where F is 50 N.

$$\sigma = \frac{50 \times 9 \cdot 81}{2298}$$

$\sigma = 2.1348*10^5\ N/m^2$.

Case2: Calculating the knee's force during stair ascent.

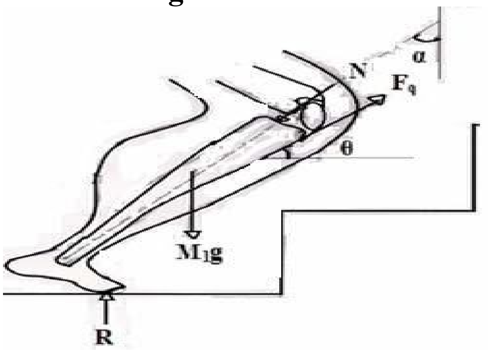

Fig 5. While ascending the stair.

In this diagram:
M1g signifies the weight of the leg, approximately equivalent to one-tenth of the total body weight.
R represents the normal reaction force from the ground.
N indicates the axial compressive force applied to the tibia (the leg bone).
Fq symbolizes the force generated by the quadriceps muscle.
α stands for the angle formed with respect to the vertical axis.
θ signifies the angle between Fq and the horizontal axis.

$\Sigma F_x = 0$

$F_q \cos\theta = N \sin\alpha$

$\Sigma F_y = 0$

$F_q \sin\theta + R = m_1 g + N \cos\alpha$

Let's take the example of a male human body weighing 50 kg, which means that m1 equals 2.1 kg. The normal reaction force R is calculated as 50 * 9.81. We found that α is equal to 40°, and the corresponding θ is 220° by substituting these values into the previously mentioned equations.

By solving that gives,
N =1937.72 N

Calculating the Normal Stress for Ascending steps

$$\sigma = \frac{F}{A}$$

$$\sigma = \frac{1937.72}{2298}$$

$\sigma = 8.43*10^5\ N/m^2$

Case3: Evaluating the contact stress experienced by the knee joint.

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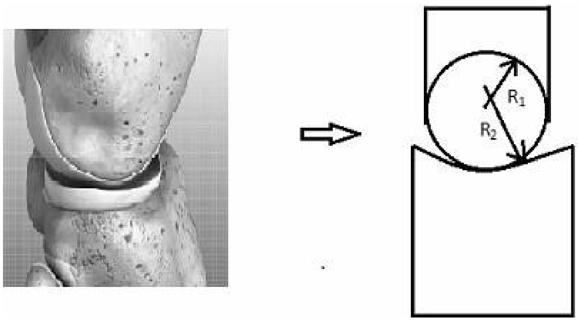

Fig 6. While in contact with different radius.

The lower section of the femur possesses a concave configuration with a radius denoted as R1, whereas the upper portion of the tibia exhibits a convex shape characterized by a radius designated as R2. It is important to note that R1 is smaller than R2.

In the flexed position of the joint, the radius of the femur, R1, measures 50 mm, while the radius of the tibia, R2, measures 100 mm, as depicted in the illustration.

Hertz contact stress in the knee joint.

Maximum contact pressure,

$$P = 0.578\sqrt[3]{\frac{F}{\Delta^2}}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right]$$

where □ = material Property,

Material property $\Delta = \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}$

In this context, v1 represents the Poisson's ratio for the femur, v2 denotes the Poisson's ratio for the tibia, E1 signifies the modulus of elasticity for the femur, and E2 stands for the modulus of elasticity for the tibia. Both the femur and tibia have a Poisson's ratio of v1 = v2 = 0.46, and their respective modulus of elasticity values are E1 = E2 = 30 MPa.

$\Delta = 5.256 \times 10^{-8}$

$$P = 0.578\sqrt[3]{\frac{490}{(5.256\times10^{-8})^2}}\left[\frac{1}{50} - \frac{1}{100}\right]$$

□= 6.9917×10^5□

V. Finite Element Analysis (FEA) of Prosthetic Knee Joint [7]

Finite Element Analysis (FEA) is a simulation technique employed to address intricate and irregular geometric shapes. It involves dividing the entire object into a finite number of smaller elements and solving differential equations to obtain results. For our prosthetic knee joint analysis, we utilized the Ansys analysis software, which is a widely used tool for this purpose.

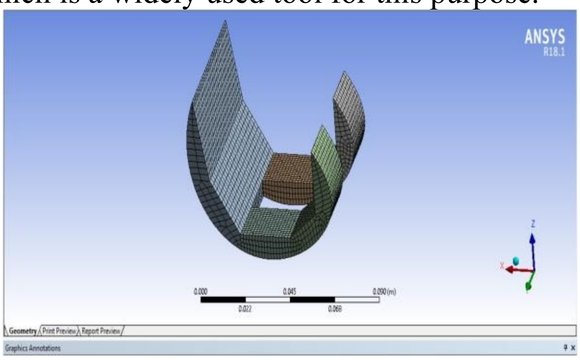

Fig 7. Meshing of model.

The provided illustration displays the mesh model of the kneecap in both the standing position and during stair ascent, subject to various loads ranging from 50 to 70. This mesh model was employed to assess the stress exerted on the knee joint.

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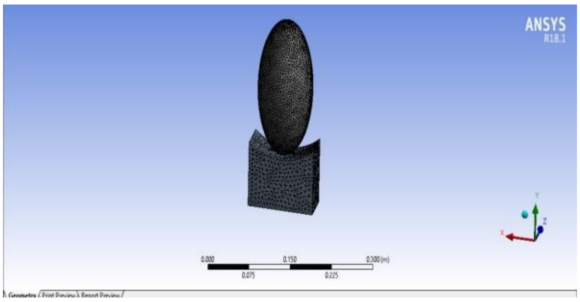

Fig 8. Meshing of the contact model.

The figure illustrates the contact mechanics of the knee joint, considering different radii of 50 and 100, while subjected to various loads. This setup allows for the determination of contact pressure variations.

VI. Result and Conversation

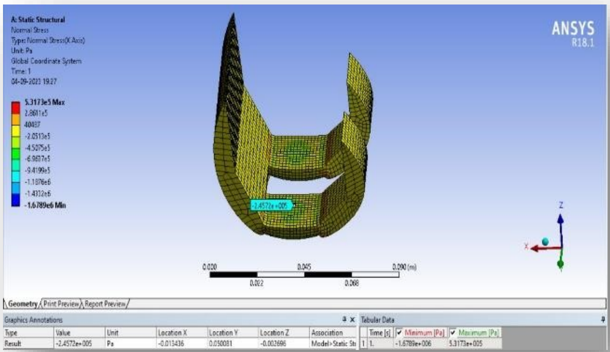

Fig 9. Stress analysis for the knee joint when a person is in a standing position.

The initial configuration entails fixing the model at the tibial portion of the knee joint and applying a central load of 50 kilograms, which equals 490 Newtons. The goal is to assess and identify the stress distribution at the central location.

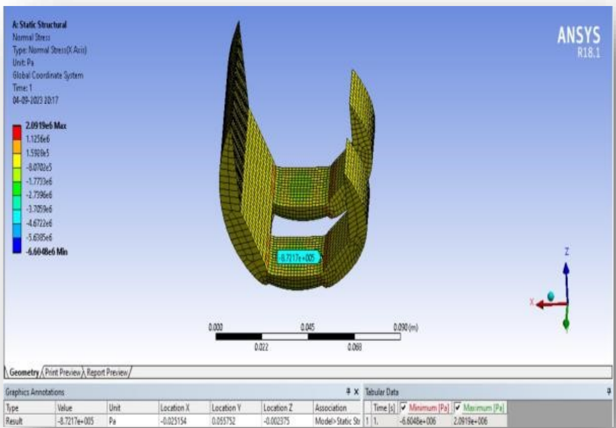

Fig 10. Stress distribution in the knee joint during stair ascent.

In this specific situation, the initial phase involves ascertaining the knee angles and computing the reaction force. Following that, a load is administered at distinct angles, precisely at 220 and 40 degrees, with the load being centred on the knee.

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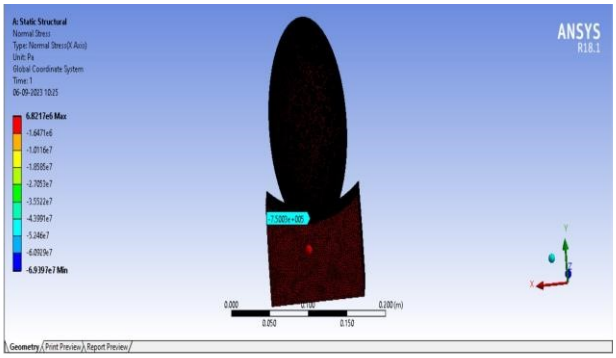

Fig 11. Hertz contact stress of both contact bodies.

In this scenario, the loads are applied at the center where the contact point is linked, and the goal is to determine the maximum pressure at the contact point while varying the loads from 50N to 70N.

Analytical vs. FEA Graph Comparison[8]

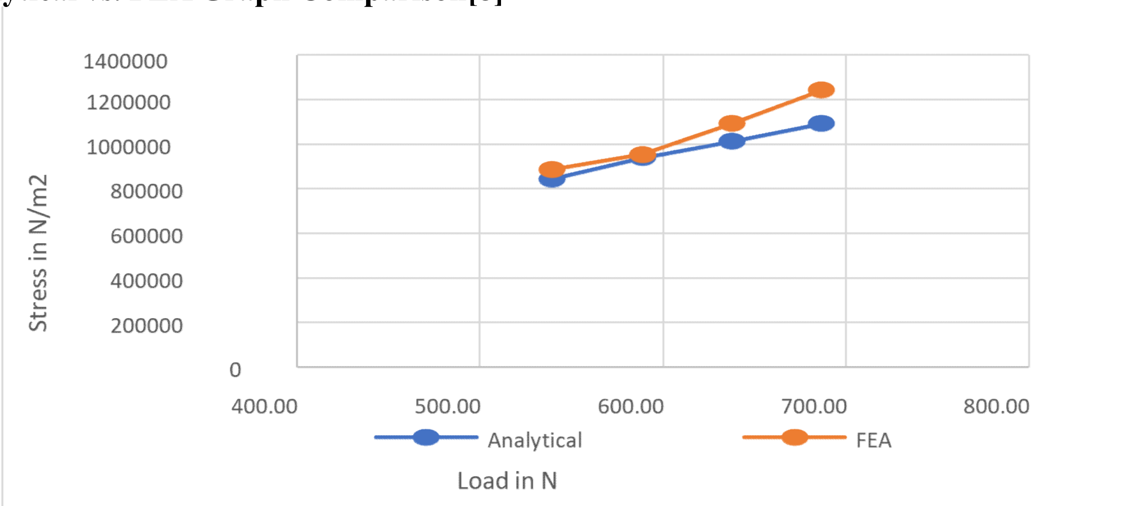

Fig 12. graph indicates that load vs stress while standing position.

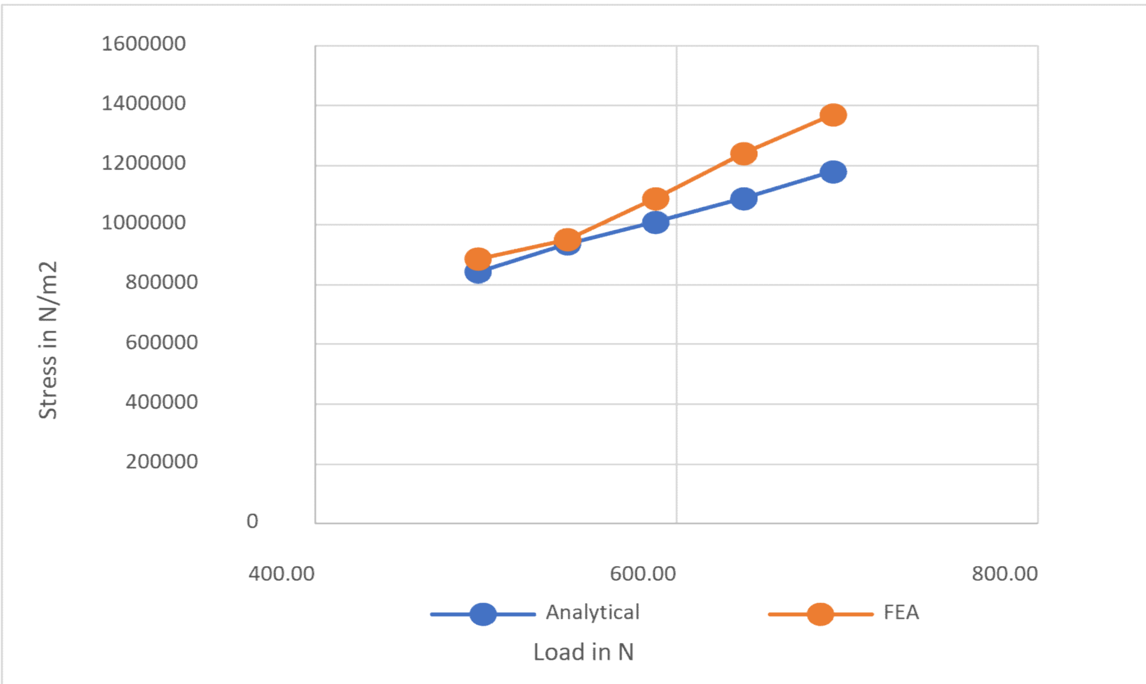

Fig 13. Graph shows that load vs stress while ascending the steps.

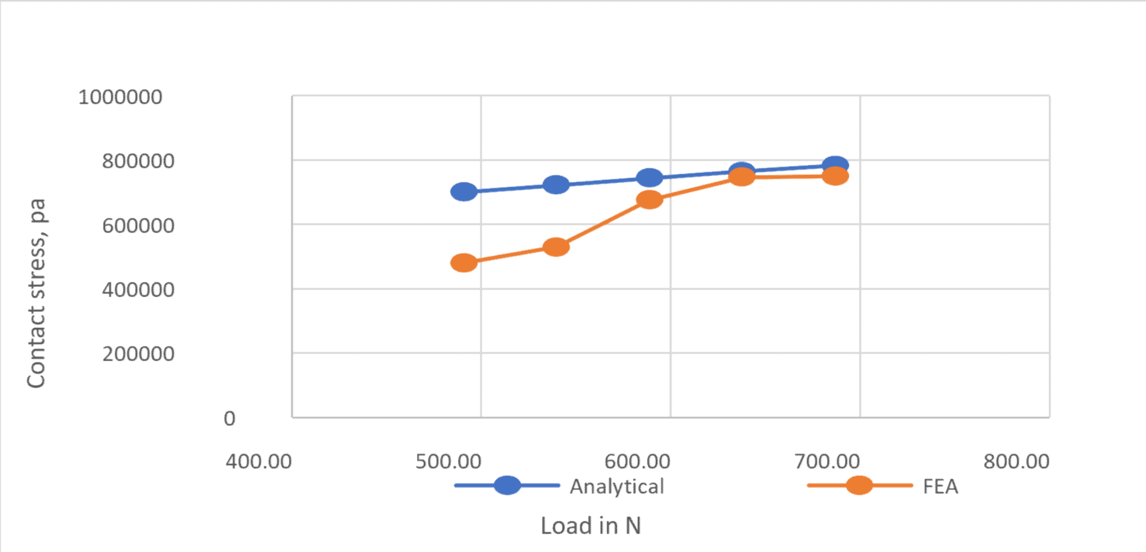

Fig 14. Graph shows that Load vs Stress of contact knee joint.

Figure 11 illustrates a graph depicting the relationship between load and stress under various loads, while in standing position including 50N, 55N, 60N, and 65N. The obtained results are then compared with both Analytical and Finite Element Analysis (FEA) outcomes. The disparity between these results is noted, with the blue data points representing the Analytical results, and the red data points representing the FEA results, facilitating the validation comparison between the two methods.

Figure 12 presents a graphical representation displaying the correlation between load and stress during stair ascent, considering loads of 50N, 55N, 60N, 65N, and 70N. The results obtained are subsequently compared with both Analytical and Finite Element Analysis (FEA) outcomes. Any discrepancies between these results are observed, where the blue data points denote the Analytical results and the red data points represent the FEA results. This comparison allows for the validation assessment of the two methods.

Figure 13 presents a graphical representation depicting the correlation between load and stress in the context of contact stress in knee joint, considering loads of 50N, 55N, 60N, 65N, and 70N. The results obtained are subsequently compared with both Analytical and Finite Element Analysis (FEA) outcomes. Any disparities between these results are observed, with the blue data points signifying the Analytical results and the red data points denoting the FEA results. This comparison aids in the validation assessment of the two methods.

VII. Conclusion

In this case, we employed CATIA software to create a design approach for the tibia and femur components. We conducted an analysis of normal and contact stress in prosthetic knee joints while subjecting them to different load conditions. Our findings indicate that the percentage difference between the Analytical and FEA results is below 10%, indicating a reasonably high level of agreement between the two methods. This study provided valuable insights into how stress is distributed within a knee joint model and how it responds to varying loads. Furthermore, it was observed that an increase in load resulted in higher levels of both normal and contact stress within the knee joint.

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